

Modified Dihadron Fragmentation Functions in Hot and Nuclear Matter

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(Dated: February 9, 2008)

Medium modification of dihadron fragmentation functions due to gluon bremsstrahlung induced by multiple partonic scattering is studied in both deep-inelastic scattering (DIS) off large nuclei and high-energy heavy-ion collisions within the same framework of twist expansion. The modified fragmentation functions for dihadrons are found to follow closely that of single hadrons leading to a weak nuclear suppression of their ratios as measured by HERMES in DIS experiments. Meanwhile, a moderate medium enhancement of the near-side correlation of two high transverse momentum hadrons with increasing centrality is found in heavy-ion collisions because of the trigger bias and the increase in parton energy loss with centrality. Successful comparisons between theory and experiment for multi-hadron observables in both confining and deconfined environments offers comprehensive evidence for partonic energy loss as the mechanism of jet modification in dense matter.

PACS numbers: 12.38.Mh, 11.10.Wx, 25.75.Dw

Medium modification of the jet structure has emerged as a new diagnostic tool for the study of partonic properties of the dense matter [1]. The modification goes beyond a mere suppression of inclusive spectra of leading hadrons and can be extended to include the modification of many particle observables, the simplest of which are two-hadron correlations within the jet cone. Such two-hadron correlations have been measured both in DIS [2] and high-energy heavy-ion collisions [3, 4]. While the two-hadron correlation is found slightly suppressed in DIS off a nucleus versus a nucleon target, it is moderately enhanced in central $Au + Au$ collisions relative to that in $p + p$. This is in sharp contrast to the observed strong suppression of single inclusive spectra [5, 6] in either DIS or central $A + A$ collisions and constitutes a necessary test of any energy loss formalism used to compute the modification of the single inclusive spectrum.

Theoretically, multi-particle correlations from jet fragmentation and their medium modification can be studied through n -hadron fragmentation functions which can be defined as the overlapping matrices of partonic field operators and n -hadron states. These n -hadron fragmentation functions are non-perturbative and involve long distance processes. However, they may be factorized from the hard perturbative processes and their evolution with momentum scale may be systematically studied in perturbative QCD (pQCD) [7], similarly as the usual single-inclusive hadron fragmentation function (SFF) [8]. In this respect, they serve as an important test of the factorized pQCD formalism currently being applied in the computation of jet-like observables.

In this Letter, we report our first study of the medium modification of dihadron fragmentation functions (DFF) in both cold nuclei and hot deconfined partonic matter. The medium modification of DFFs in DIS off nuclei will be derived within the framework of generalized factor-

ization and twist expansion [9, 10]. The results are then extended to the case of parton propagation in heavy-ion collisions. With exactly the same parameters determined from the modified SFFs [10, 11], the medium modification of two-hadron correlations in both DIS off nuclei and heavy-ion collisions are predicted and compared to the experimental data.

Similar to SFFs, the DFF of a quark into two hadrons of flavor h_1, h_2 , with total forward momentum $p_h^+ \equiv p_1^+ + p_2^+$ (total momentum fraction $z \equiv z_1 + z_2$) and relative transverse momentum $q_\perp = p_{1\perp} - p_{2\perp}$ can be defined as the Fourier transform of the overlapping matrices of the quark fields $\psi_q(x), \bar{\psi}_q(0)$ and two-hadron inclusive final states $|p_1, p_2, S - 2\rangle$ as,

$$D_q^{h_1, h_2}(z_1, z_2) = \frac{z^4}{4z_1 z_2} \int \frac{d^2 q_\perp}{4(2\pi)^3} \int \frac{d^4 p}{(2\pi)^4} \times \delta\left(z - \frac{p_h^+}{p^+}\right) \text{Tr} \left[\frac{\gamma^+}{2p_h^+} \int d^4 x e^{ip \cdot x} \sum_{S=2} \times \langle 0 | \psi_q(x) | p_1, p_2, S - 2 \rangle \langle p_1, p_2, S - 2 | \bar{\psi}_q(0) | 0 \rangle \right], \quad (1)$$

and can be factorized from the hard processes [7]. They also satisfy the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations as derived in Ref. [7]. One unique feature of these equations is the contribution from independent fragmentation of two partons after the parton splitting. These equations have been solved numerically [7] and the Q^2 evolution agrees very well with results from JETSET [12] Monte Carlo simulations of $e^+ + e^- \rightarrow h_1 + h_2 + X$ processes. Although both the single and dihadron fragmentation functions evolve rapidly with Q^2 , their ratio has a very weak Q^2 dependence. In the absence of experimental data, JETSET Monte Carlo results will be used as the initial condition for the vacuum DFF in this study. For SFFs, the BKK

parameterization [13] will be used; which also agrees well with JETSET results.

Applying factorization to dihadron production in single jet events in DIS off a nucleus, $e(L_1) + A(p) \rightarrow e(L_2) + h_1(p_1) + h_2(p_2) + X$, one can obtain the dihadron semi-inclusive cross section,

$$E_{L_2} \frac{d\sigma_{\text{DIS}}^{h_1 h_2}}{d^3 L_2 dz_1 dz_2} = \frac{\alpha^2}{2\pi s} \frac{1}{Q^4} L_{\mu\nu} \frac{dW^{\mu\nu}}{dz_1 dz_2}, \quad (2)$$

in terms of the semi-inclusive tensor at leading twist,

$$\begin{aligned} \frac{dW^{\mu\nu}}{dz_1 dz_2} &= \sum_q \int dx f_q^A(x, Q^2) H^{\mu\nu}(x, p, q) \\ &\times D_q^{h_1, h_2}(z_1, z_2, Q^2). \end{aligned} \quad (3)$$

In the above, $L_{\mu\nu} = (1/2) \text{Tr}(\not{L}_1 \gamma_\mu \not{L}_2 \gamma_\nu)$, the factor $H^{\mu\nu}$ represents the hard part of quark scattering with a virtual photon which carries a four-momentum $q = [-Q^2/2q^-, q^-, \vec{0}_\perp]$ and $f_q^A(x, Q^2)$ is the quark distribution in the nucleus which has a total momentum $A[p^+, 0, \vec{0}_\perp]$. The hadron momentum fractions, $z_1 = p_1^-/q^-$ and $z_2 = p_2^-/q^-$, are defined with respect to the initial momentum q^- of the fragmenting quark.

At next-to-leading twist [9, 10], the dihadron semi-inclusive tensor receives contributions from multiple scattering of the struck quark off soft gluons inside the nucleus with induced gluon radiation. One can reorganize the total contribution (leading and next-to-leading twist) into a product of effective quark distribution in a nucleus, the hard part of photon-quark scattering $H^{\mu\nu}$ and a modified DFF: $\tilde{D}_q^{h_1, h_2}(z_1, z_2)$ (which includes, within it, the effect of subsequent scattering and gluon radiation). The calculation of the modified DFF at the next-to-leading twist in a nucleus proceeds [14] similarly as that for the modified SFFs [10] and yields,

$$\begin{aligned} \tilde{D}_q^{h_1, h_2}(z_1, z_2) &= D_q^{h_1, h_2}(z_1, z_2) + \int_0^{Q^2} \frac{dl_\perp^2}{l_\perp^2} \frac{\alpha_s}{2\pi} \\ &\times \left[\int_{z_1+z_2}^1 \frac{dy}{y^2} \left\{ \Delta P_{q \rightarrow qg}(y, x_B, x_L, l_\perp^2) D_q^{h_1, h_2}\left(\frac{z_1}{y}, \frac{z_2}{y}\right) \right. \right. \\ &\quad \left. \left. + \Delta P_{q \rightarrow gq}(y, x_B, x_L, l_\perp^2) D_g^{h_1, h_2}\left(\frac{z_1}{y}, \frac{z_2}{y}\right) \right\} \right. \\ &\quad \left. + \int_{z_1}^{1-z_2} \frac{dy}{y(1-y)} \Delta \hat{P}_{q \rightarrow qg}(y, x_B, x_L, l_\perp^2) \right. \\ &\quad \left. \times D_q^{h_1}\left(\frac{z_1}{y}\right) D_g^{h_2}\left(\frac{z_2}{1-y}\right) + (h_1 \rightarrow h_2) \right]. \end{aligned} \quad (4)$$

In the above, $x_B = -Q^2/2p^+q^-$, $x_L = l_\perp^2/2p^+q^-y(1-y)$, l_\perp is the transverse momentum of the radiated gluon, $\Delta P_{q \rightarrow qg}$ and $\Delta P_{q \rightarrow gq}$ are the modified splitting functions with momentum fraction y , whose forms are identical to that in the modified SFF [10]. The switch $(h_1 \rightarrow h_2)$ is only meant for the last term, which represents independent fragmentation of the quark and radiated gluon. The

corresponding modified splitting function,

$$\Delta \hat{P}_{q \rightarrow qg} = \frac{1+y^2}{1-y} \frac{C_A 2\pi\alpha_s T_{qg}^A(x_B, x_L)}{(l_\perp^2 + \langle k_\perp^2 \rangle) N_c f_q^A(x_B, Q^2)}, \quad (5)$$

is similar to $\Delta P_{q \rightarrow qg}$ but does not contain contributions from virtual corrections. In the above, $C_A = N_c = 3$, and $\langle k_\perp^2 \rangle$ is the average intrinsic parton transverse momentum inside the nucleus.

Note that both modified splitting functions depend on the quark-gluon correlation function T_{qg}^A in the nucleus that also determines the modification of SFFs [10]. For a Gaussian nuclear distribution, it can be estimated as,

$$T_{qg}^A(x_B, x_L) = \tilde{C}(Q^2) m_N R_A f_q^A(x_B) (1 - e^{-x_L^2/x_A^2}), \quad (6)$$

where, $x_A = 1/m_N R_A$, m_N is the nucleon mass and $R_A = 1.12A^{1/3}$ is the nuclear radius. The small variation of the gluon correlation function is neglected; its average value is absorbed into the overall constant \tilde{C} . This is the only parameter in the modified DFF which might depend on the kinematics of the process but is identical to the parameter in the modified SFF. In the phenomenological study of the SFFs in DIS off nuclei, $\tilde{C} = 0.006 \text{ GeV}^2$ was determined within the kinematics of the HERMES experiment [5]. The predicted dependence of nuclear modification of the SFF on the momentum fraction z , initial quark energy $\nu = q^-$ and the nuclear size R_A agrees very well with the HERMES experimental data [11].

With no additional parameters in Eq. (4), one can predict the nuclear modification of DFFs within the same kinematics. Since the DFFs are connected to SFFs via sum rules [7], it is more illustrative to study the modification of the distribution for the second rank hadrons normalized by the number of leading hadrons (*i.e.*, hadrons with $z > 0.5$),

$$N_{2h}(z_2) \equiv \int_{0.5}^{1-z_2} dz_1 D_q^{h_1, h_2}(z_1, z_2) \bigg/ \int_{0.5}^1 dz_1 D_q^{h_1}(z_1), \quad (7)$$

where z_1 and $z_2 < z_1$ are the momentum fractions of the triggered (leading) and associated (secondary) hadrons, respectively. Shown in Fig. 1 is the predicted ratio of the normalized associated hadron distribution in DIS off a Nitrogen ($A = 14$) and Krypton ($A = 84$) target to that off a proton ($A = 1$) *i.e.*, $R_{2h}(z_2) = N_{2h}^A(z_2)/N_{2h}^1(z_2)$, as compared to the HERMES experimental data [2].

The agreement between the prediction and the data is remarkable given that no free parameters are used. The suppression of $R_{2h}(z_2)$ at large z_2 with atomic number, is quite small compared to the suppression of the SFFs [5, 11]. Since $N_{2h}(z_2)$ is the ratio of double and single hadron fragmentation functions, the effect of induced gluon radiation or quark energy loss is mainly borne by the single spectra of the leading hadrons. At small values of z_2 , the modified DFF rises above its vacuum counterpart more than the modified SFF. This is due to the

new contribution where each of the detected hadrons emanates from the independent fragmentation of the quark and the radiated gluon. In the experiment, the measured ν and Q^2 vary with z_1 and z_2 ; this has been incorporated in the calculation

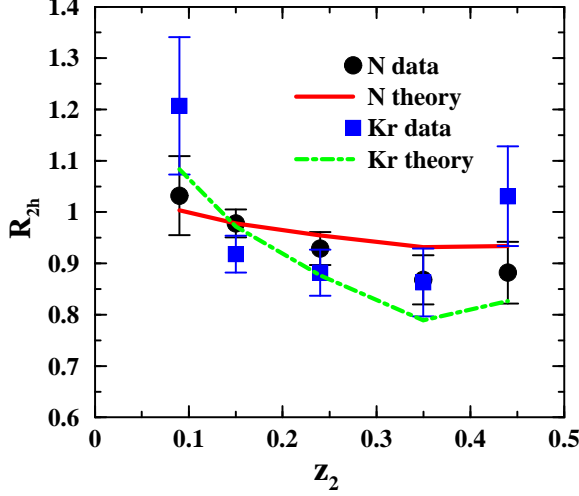


FIG. 1: (Color online) Results on the medium modification of the associated hadron distribution [$R_{2h}(z_2) = N_{2h}^A(z_2)/N_{2h}^1(z_2)$, see Eq. (7)] in nuclei versus its momentum fraction as compared with HERMES data [2] in DIS with Nitrogen (N) and Krypton (Kr) targets.

In high-energy heavy-ion (or $p+p$ and $p+A$) collisions, jets are always produced in back-to-back pairs. Correlations of two high- p_T hadrons in azimuthal angle generally have two Gaussian peaks [3, 4]. Relative to the triggered hadron, away-side hadrons come from the fragmentation of the away-side jet and are related to SFFs. On the other hand, near-side hadrons come from the fragmentation of the same jet as the triggered hadron and therefore are related to DFFs.

The near-side correlation, background subtracted and integrated over the azimuthal angle [15, 16], can be related to the associated hadron distribution or the ratio of the DFF to the SFF, both averaged over the initial jet energy weighted with the corresponding parton production cross sections. We report on the first such calculation in this Letter. Assuming a factorization of initial and final state effects (as done in the case of single inclusive observables [11, 17]), the differential cross-section for the production of two high p_T hadrons at midrapidity from the collision of two nuclei A and B at an impact parameter b between b_{min}, b_{max} is given as,

$$\begin{aligned} \frac{d\sigma^{AB}}{dy dp_T^{\text{trig}} dp_T^{\text{assoc}}} &= \int_{b_{min}}^{b_{max}} d^2b \int d^2r t_A(\vec{r} + \vec{b}/2) t_B(\vec{r} - \vec{b}/2) \\ &\times 2K \int dx_a dx_b G_a^A(x_a, Q^2) G_b^B(x_b, Q^2) \\ &\times \frac{d\hat{\sigma}_{ab \rightarrow cd}}{d\hat{t}} \tilde{D}_c^h(z_1, z_2, Q^2), \end{aligned} \quad (8)$$

where, $G_a^A(x_a, Q^2)[G_b^B(x_b, Q^2)]$ represents the nuclear parton distribution function for a parton $a(b)$ with momentum fractions $x_a(x_b)$ in a nucleus $A(B)$, $t_A(t_B)$ represents the nuclear thickness function and $d\hat{\sigma}_{ab \rightarrow cd}/d\hat{t}$ represents the hard parton cross section with Mandelstam variable \hat{t} . The final state momentum fractions z_1, z_2 represent the the momentum fractions of the two detected hadrons with respect to the initial jet energy. The factor $K \simeq 2$ accounts for higher order corrections (identical to that used in the case of the single inclusive spectra). The medium modified DFF may be expressed as in Eq. (4), with the modified splitting functions generalized from Eq. (5) as,

$$\begin{aligned} \Delta P_{q \rightarrow i} &= P_{q \rightarrow i}(y) 2\pi\alpha_s C_A T^M(\vec{b}, \vec{r}, x_a, x_b, y, l_\perp) \\ &\times \left[l_\perp^2 N_c t_A(\vec{r} + \vec{b}/2) t_B(\vec{r} - \vec{b}/2) \right. \\ &\times \left. G_a^A(x_a) G_b^B(x_b) \frac{d\hat{\sigma}}{d\hat{t}} \right]^{-1} + v.c. \end{aligned} \quad (9)$$

As in the case of DIS, l_\perp represents the transverse momentum of the radiated gluon and T^M replaces the factor T_{qg}^A (in Eq. (5)) and represents the parton-gluon correlation function in a quark-gluon plasma. The primary difference between the denominators of Eqs. (5) and (9), lies in the weighting of a particular jet like parton by the initial production process (see Ref. [18] for the evaluation of T^M in single inclusive observables).

At present, there exists no measurement of high momentum hadrons associated with a high $p_T^{\text{trig}} (> 8 \text{ GeV})$ hadron in the case of $p+p$ collisions. Therefore, we will compare the associated yields in $d+Au$ and $Au+Au$ collisions as a function of centrality as well as the associated p_T . These yields are very sensitive to the number of flavors detected. In Fig. 2, the yield of charged hadrons with associate transverse momentum in two ranges ($6 \text{ GeV} < p_T^{\text{assoc}} < p_T^{\text{trig}}$ and $4 \text{ GeV} < p_T^{\text{assoc}} < 6 \text{ GeV}$), associated with a hadron with $p_T^{\text{trig}} > 8 \text{ GeV}$, as measured in Ref. [19], are presented along with three different calculations using Eqs. (8) and (9). The dashed lines correspond to the case of all charged hadrons where as the dot-dashed lines corresponds to the case of charged pions only. In both cases, the vacuum DFFs are estimated from JETSET Monte Carlo simulations (see Ref. [7]). Unlike in the case of the experimental measurements, no decay corrections have been introduced in these calculations: the number of pairs measured, includes not only those produced directly from the fragmentation of a jet but also decay products of particles from fragmentation. As a result, such fragmentation functions tend to be somewhat larger at lower momentum fractions than fragmentation functions with such corrections. The integrated (over transverse momentum) associated yield being dominated by lower momentum fractions shows a larger effect as lower momentum ranges in p_T^{assoc} are chosen.

A decay corrected DFF [$\bar{D}(z_1, z_2)$] may be constructed, phenomenologically, by comparing with the differential spectrum of associated particles in $d+Au$ collisions (as a function of $z_T = p_T^{\text{trig.}}/p_T^{\text{assoc.}}$, in Ref. [19]):

$$\bar{D}(z_1, z_2) = 0.55 \times D(z_1, z_2)[1 + 1.4(z_1/z_2 - 1)]. \quad (10)$$

Using the above DFF, we obtain a very good agreement with the data (solid lines). All estimates show a mild rise with centrality which originates solely from the increased trigger bias as the centrality of the collision is increased. We have focused on large $p_T^{\text{trig.}}$ and $p_T^{\text{assoc.}}$, as large momenta ensure the validity of the independent fragmentation picture. Otherwise, other non-perturbative and higher twist effects such as recombination [20] and the influence of radial and longitudinal flow can become important and may lead to further modification of the associated yield [21]. As may be noted from Fig. 2, lowering the $p_T^{\text{assoc.}}$ results in a systematic departure between prediction and data with increasing centrality.

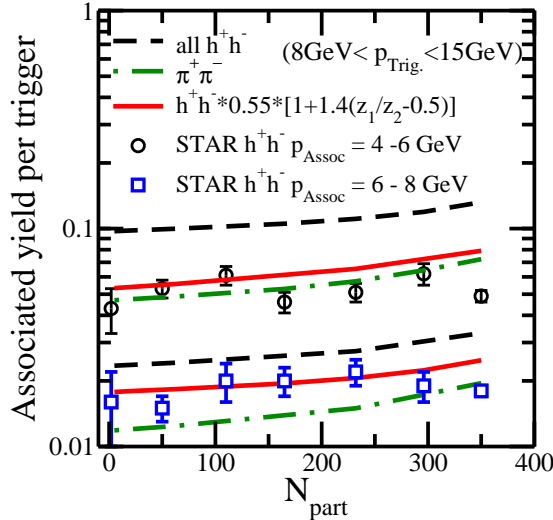


FIG. 2: (Color online) Yields of different flavors of hadrons with $6 \text{ GeV} < p_T^{\text{assoc.}} < p_T^{\text{trig.}}$ and $4 \text{ GeV} < p_T^{\text{assoc.}} < 6 \text{ GeV}$, associated with a trigger hadron with $p_T^{\text{trig.}} > 8 \text{ GeV}$ versus the centrality of $Au+Au$ collisions at $\sqrt{s} = 200 \text{ GeV}$ as compared to experimental data [19], see text for details.

In summary, we have studied the modification of the DFFs in both confining and hot deconfined matter due to multiple parton scattering and induced gluon radiation. The modification, follows closely that of SFFs so that the associated hadron distributions or the ratios of DFFs to SFFs are only slightly suppressed in DIS off nuclei but enhanced in central heavy-ion collisions due to trigger bias. With no extra parameters, our calculations agree very well with experimental data. Such calculations, combined with previous successful comparisons in the single inclusive sector [11] in both cold nuclear and hot deconfined matter constitutes a very important test of the partonic jet modification formalism and testifies to

the applicability of perturbatively calculable jet observables as probes of the dense matter created in heavy-ion collisions.

We thank F. Wang for discussions. This work was supported by the U.S. Department of Energy under Contract No. DE-AC03-76SF00098, under grant DE-FG02-05ER41367, NSFC under project Nos. 10475031 and 10135030 and NSERC of Canada.

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